Week 2
Risk, Uncertainty, and Insurance

1. Basic Concepts

(i) ***Gamble:*** An action with more than one possible outcome, such that with each outcome there is an associated probability of that outcome occurring. If the outcomes are good (G) and bad (B), denote the associated probabilities by p_G and p_B

(ii) ***Payoff:*** With each outcome is associated a "pay-off" which can be expressed in terms of money: $\$c_G$ and $\$c_B$.

(iii) ***Utility from a Payoff:*** With each payoff is associated a "utility", u(c): u(c_G) is the utility in the good situation u(c_B) is the utility in the bad situation. We assume that: $u'(c) = du/dc > 0$

(iv) ***Expected Return:*** The expected return from the gamble is:
$$ER = p_G c_G + p_B c_B$$

(v) ***Expected Utility:*** The expected utility from the gamble is:
$$EU = p_G u(c_G) + p_B u(c_B)$$

(vi) ***Fair Gamble:*** A fair gamble is one in which the sum that is bet (*W*) is equal to the expected return.

Example: You have an initial wealth of \$500. You are offered a gamble:
\$250 with $\pi_B = 0.5$ or \$750 with $\pi_G = 0.5$
You can accept the gamble or you can decline the gamble
If you decline you keep \$500 with certainty: u(500); If you accept: EU= 0.5*u(250) +0.5*u(750)
This gamble is called a *fair gamble* because the amount that is bet (\$500) is equal to the expected return the gamble (£500).
You will *reject* the gamble if u(500)>EU; You will *be indifferent* to the gamble if u(500)=EU;
You will *accept* the gamble if u(500)<EU;

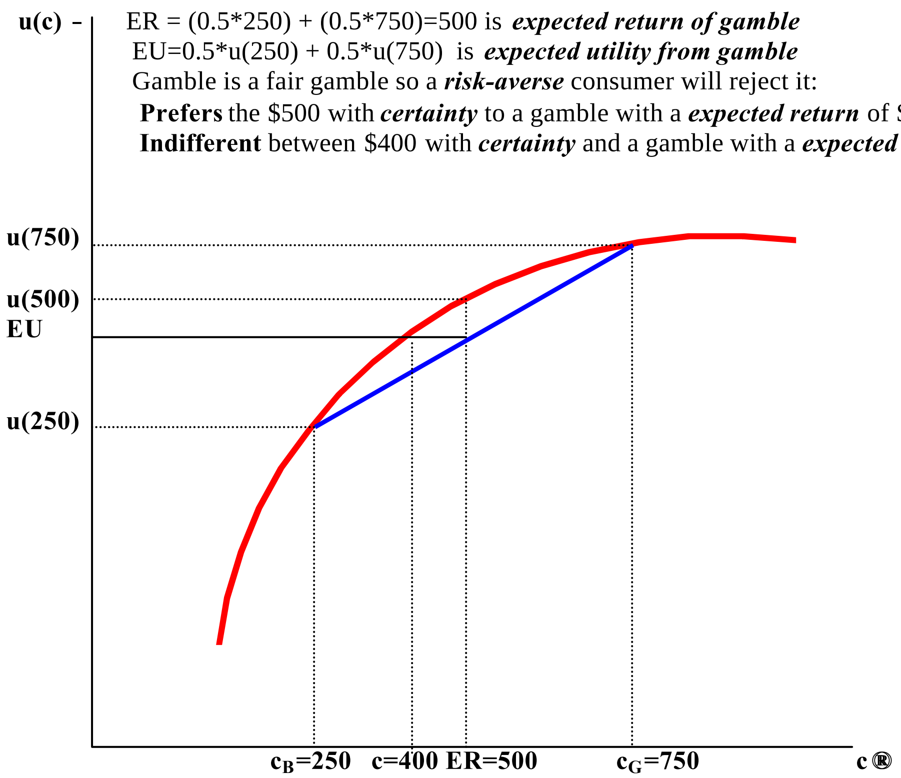
2. Attitudes to Risk

A *risk averse* person will never accept a fair gamble
A *risk-indifferent person* will be indifferent between not gambling and a fair gamble
A *risk loving* person will always accept a fair gamble

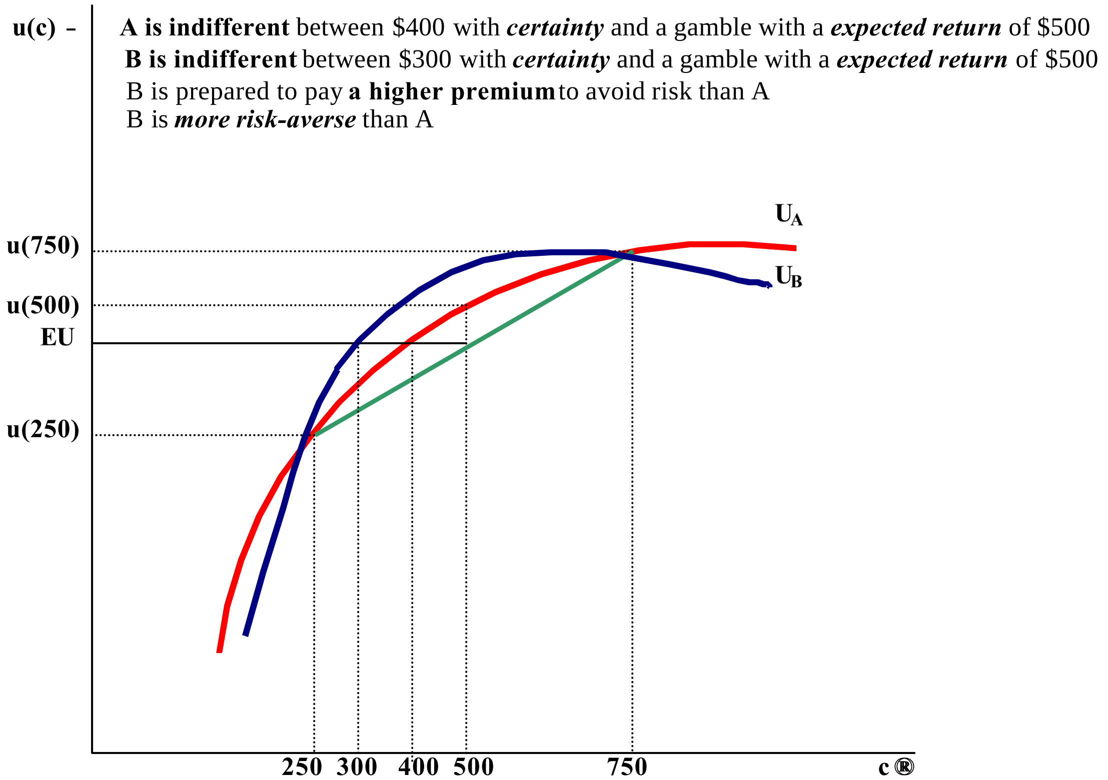
The following statements are equivalent:
A person is risk-averse; A person's utility function is concave; A person's marginal utility of income diminishes with income.

The following statements are equivalent:
A person is risk-indifferent; A person's utility function is linear; A person's marginal utility of income is constant.

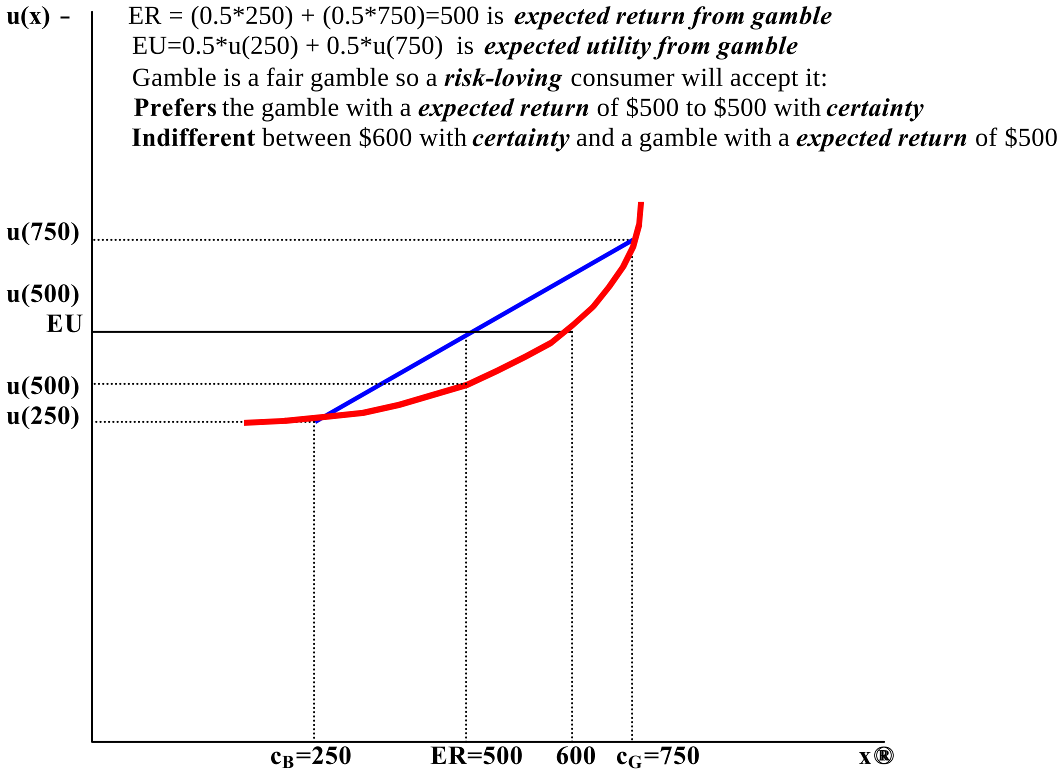
The following statements are equivalent:
A person is a risk-lover; A person's utility function is convex; A person's marginal utility of income increases with income.



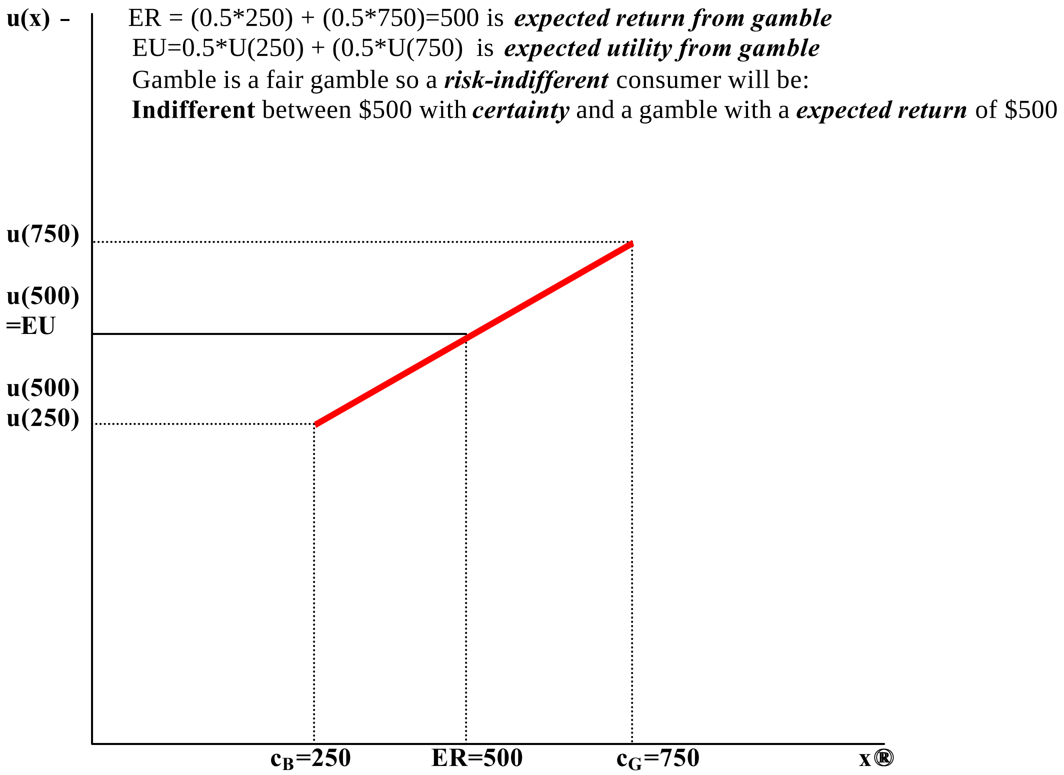
Risk Aversion and a Concave Utility Function



Degrees of Risk-Aversion



Risk Loving Behaviour and a Convex Utility Function



Risk Indifference and a Linear Utility Function

	Poor Rainfall (p=0.5)	*Good Rainfall (p=0.5)*
Low effort (e=0)	\$10,000	\$20,000
High effort (e=1)	\$20,000	\$40,000

Now go back to the example of the previous week. Putting in high effort in work is to accept a *gamble* with ER=(w_0+ w_1)/2 –c_1. On the other hand, to put in low effort is to reject the gamble and so receive w_0-c_0 with *certainty*. The worker would decide whether to accept or reject the gamble by comparing: u(w_0-c_0) with EU=[u(w_1-c_1)+u(w_0-c_1)]/2

If the worker is risk averse and (w_0+ w_1)/2 –c_1= w_0-c_0 (that is, it is a *fair* gamble) he will reject the gamble and not put in the effort.

So to induce him to accept the gamble, ER=(w_0+ w_1)/2 –c_1>w_0-c_0. This means that w_1, the payoff in the good situation must be sufficiently high (we assume w_0 is a subsistence/minimum wage that can't be lowered).

How high w_1 needs to be depends on worker's *aversion to ri*

Risk and Uncertainty

The problem of risk and uncertainty arises when goods are distinguished by 'time of consumption' and by 'state of nature' (or *contingency*). The assumption that efficient markets exist for goods under all contingencies, and at all times, assumes that consumers can buy insurance on actuarially fair terms so that, regardless of contingency, utility remains unchanged.

The Demand For Insurance

Suppose a consumer has a wealth of \$W and there is some probability π that he will lose *\$X* in an adverse contingency. The consumer can buy insurance that will pay him *\$Z* in the event of this contingency and the price of insurance is γ per \$ of cover. To find out how much insurance the consumer will purchase, formulate his problem as one of maximising his *expected utility*:

$$\underset{Z}{Max}\ EU(Z) = \pi U(W - X - \gamma Z + Z) + (1-\pi)U(W - \gamma Z) \qquad (1)$$

The first order condition for solving this problem is:

$$EU'(Z) = \pi U'[W - X - (1-\gamma)Z](1-\gamma) - (1-\pi)U'[W - \gamma Z]\gamma = 0 \qquad (2)$$

which simplifies to:

$$\frac{p}{1-p}\frac{U'[W-X-(1-g)Z]}{U'[W-gZ]}=\frac{g}{1-g} \tag{3}$$

If the insurance company charged an actuarially fair premium (γ=π), the consumer would be fully insured since:

$$\begin{aligned} &U'[W-X-(1-g)Z]=U'[W-gZ] \\ &\Rightarrow W-X-(1-g)Z=W-gZ\Rightarrow X=Z \end{aligned} \tag{4}$$

The expected profit of the insurance company is:

$$\begin{aligned} &p(gZ-Z)+(1-p)gZ \\ &=(g-p)Z \end{aligned} \tag{5}$$

and so, if the insurance industry is competitive, profits would be competed away and the insurance company would charge an actuarially fair premium (γ=π). Conversely, if barriers to entry meant that there competition in the insurance industry was imperfect, then insurance firms would continue to make supernormal profits (with γ > π) and there would be 'market failure' due to the fact that consumers would not be fully insured (see Figure 1, below).

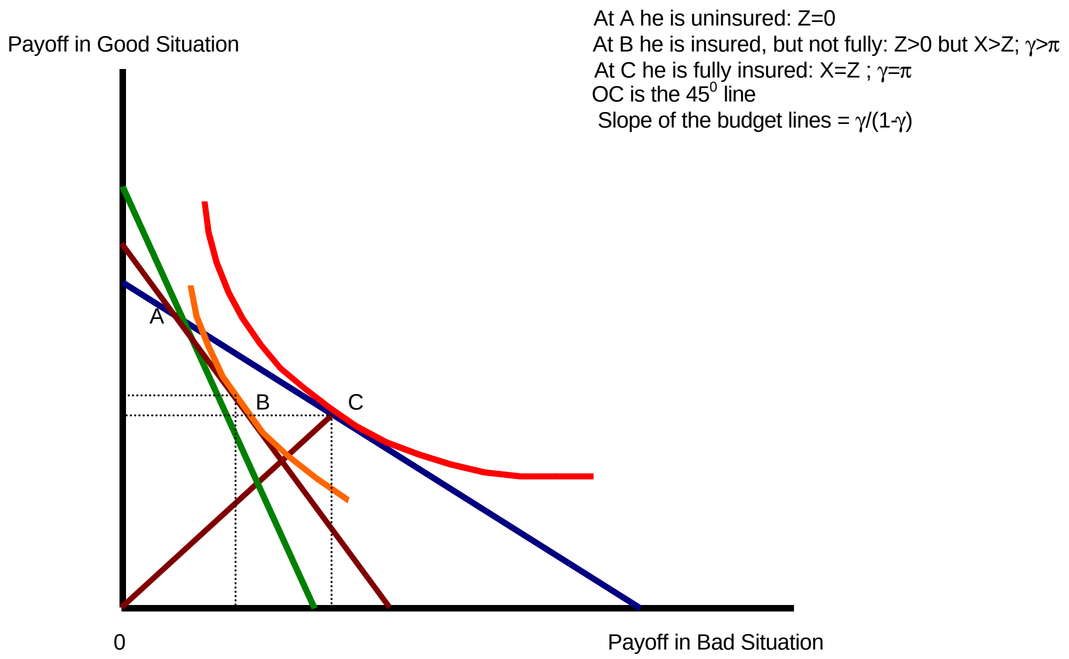


Figure 1:
The Demand for Insurance