Week 5
Public Goods and Voting Mechanisms

1. Types of Public Goods

Private goods are *rivalrous* (a unit of a good consumed by one person cannot also be consumed by another person) and *excludable* (a person who does not pay for a good can be excluded from its consumption) in consumption. By contrast, a **pure public good** is *non-rivalrous* (a given amount of the good can be consumed by one person without affecting its simultaneous consumption by another) and *non-excludable* (non-payment does not entail exclusion from consumption). Within these poles of non-rivalrousness and non-excludability, **impure public goods** represent in between cases. Impure public goods arise because of *congestion costs*: the value to existing users of a public good falls as more users are added. Impure public goods are, therefore, *partially rivalrous*. Within this category of impure public goods, it is possible to distinguish between:

(i) **Common Property Resources:** public goods subject to congestion from which exclusion is not possible

(ii) **Club goods:** public goods subject to congestion from which exclusion is possible

(i) **Variable Use goods:** public goods subject to congestion where the amount of services used by consumers can be varied

2. The Efficient Provision of a Discrete (Pure) Public Good

There are two goods - a 'private' good and a 'public' good – and two persons, indexed $i=1,2$. The public good is either provided ($G=1$) or not provided ($G=0$) and the cost of providing it is C; the private good, on the other hand, is supplied in varying quantities. W_i is the wealth of the agent i and X_i is his expenditure on the private good and G_i is his contribution towards the provision of the public good. Consequently:

$$G\begin{cases} = 0 \text{ if } G_1 + G_2 < C \\ = 1 \text{ if } G_1 + G_2 \geq C \end{cases} \tag{1}$$

If $U_1(X_1, G)$ and $U_2(X_2, G)$ are the utility functions of the two persons, then it will be efficient to provide the public good[1] (*G=1*) if for some pattern of contributions (G_1, G_2), such that $G_1+G_2 \geq C$:

$$U_i(W_i - G_i, 1) \geq U_i(W_i, 0) \ \forall i = 1,2 \tag{2}$$

with the strict inequality, >, holding for at least one *i*.

Define the reservation price of consumer *i* for the public good (which is the consumer's *maximum* willingness-to-pay for the public good) as R_i where:

$$U_i(W_i - R_i, 1) = U_i(W_i, 0) \tag{3}$$

Then the necessary and sufficient conditions for the provision of the public good to be Pareto improving are:

$$R_i > G_i, i = 1,2 \text{ and } \sum_i^2 R_i > \sum_i^2 G_i \geq C \tag{4}$$

How effective would the market be at providing the public good? Suppose that $R_i=100$, $i=1,2$ and $C=150$ so that by equation (4), it is efficient to provide the public good.

		Consumer 2	
		Buy	Don't Buy
Consumer 1	Buy	-50,-50	-50,100
	Don't Buy	100,-50	0,0

Source: Varian (1992), p. 417.

Each consumer has to decide independently whether or not to buy the public good: if consumer 1 buys, then 2 can *free ride* by not buying; similarly, if consumer 2 buys, then 1 can *free ride* by not buying. Therefore, as in the above table of payoffs, the dominant strategy is for both consumers to *not buy* the public good[2]. So the net result is that the good is not provided even though it would be efficient to do so.

3. The Efficient Provision of the Quantity of a (Pure) Public Good

When the quantity of the public good can be varied, let G denote the quantity of the public good (where *G=0* implies no provision) and let *C=C(G)* denote the cost of provision. Then an efficient combination of the amounts of the private and the public

[1] Or, conversely, it will be inefficient to not provide the public good (G=0).
[2] This game has a structure similar to that of the *Prisoner's Dilemma*.

good will be produced when one consumer's utility is maximised, subject to the other consumer's utility being fixed at some level and subject to the budget constraint[3]:

$$\underset{X_1,X_2,G}{Max}\ U_1(X_1,G) \text{ s.t. } U_2(X_2,G)=\bar{U}_2 \text{ and } X_1+X_2+C(G)=W_1+W_2 \qquad (5)$$

This yields the equilibrium condition:

$$\frac{\partial U_1(X_1,G)/\partial G}{\partial U_1(X_1,G)/\partial X_1}+\frac{\partial U_2(X_2,G)/\partial G}{\partial U_2(X_2,G)/\partial X_2}=C'(G) \qquad (6)$$

or, in other words:

$$MRS^1_{XG}+MRS^2_{XG}=MC_G \qquad (7)$$

The equilibrium condition in equation (7) is known as the *Samuelson Condition* (Samuelson, 1954) for the efficient provision of a (pure) public good and is illustrated in Figure 13, below. (This condition is derived in the Mathematical Appendix). For two consumers, A and B, the optimal production is O_AX_0 of the private good and O_AG_0 of the public good. The optimal distribution of the private good (so that A is kept on the indifference curve II) is O_AZ to A and O_AB to B. At the point of tangency between TT and JJ we have: $MRS^B{}_{GX} + MRS^A{}_{GX} = MRT_{GX}$.

Example. Suppose C(G)=G and the utility functions are Cobb-Douglas so that $U_i(X_i,G)=\log X_i+\beta_i\log G$. Then $MRS^i_{XG}=(\partial U_i/\partial G)/(\partial U_i/\partial X_i)=\beta_i X_i/G$ and the efficiency condition is: $\sum\beta_i X_i/G=1 \Rightarrow G=\sum\beta_i X_i$. Using the constraint $\sum X_i+G=W$ defines the set of efficient allocations: $(G,X_1..X_N)$. Note that there may be many levels of efficient provision of the public good.

Example: Suppose C(G)=G and the utility functions are quasi-linear[4] so that $U_i(X_i,G)=X_i+\beta_i\log G$. Then $MRS^i_{XG}=(\partial U_i/\partial G)/(\partial U_i/\partial X_i)=\beta_i/G$ and the efficiency condition is: $\sum\beta_i/G=1\Rightarrow G=\sum\beta_i$ so that there is a *unique* efficient level of provision of the public good.

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[3] See Varian (2003), p. 666.

[4] For a quasi-linear utility function: $U(X_i,G)=X_i+V_i(G)$. This implies that the marginal utility of the private good is always 1.

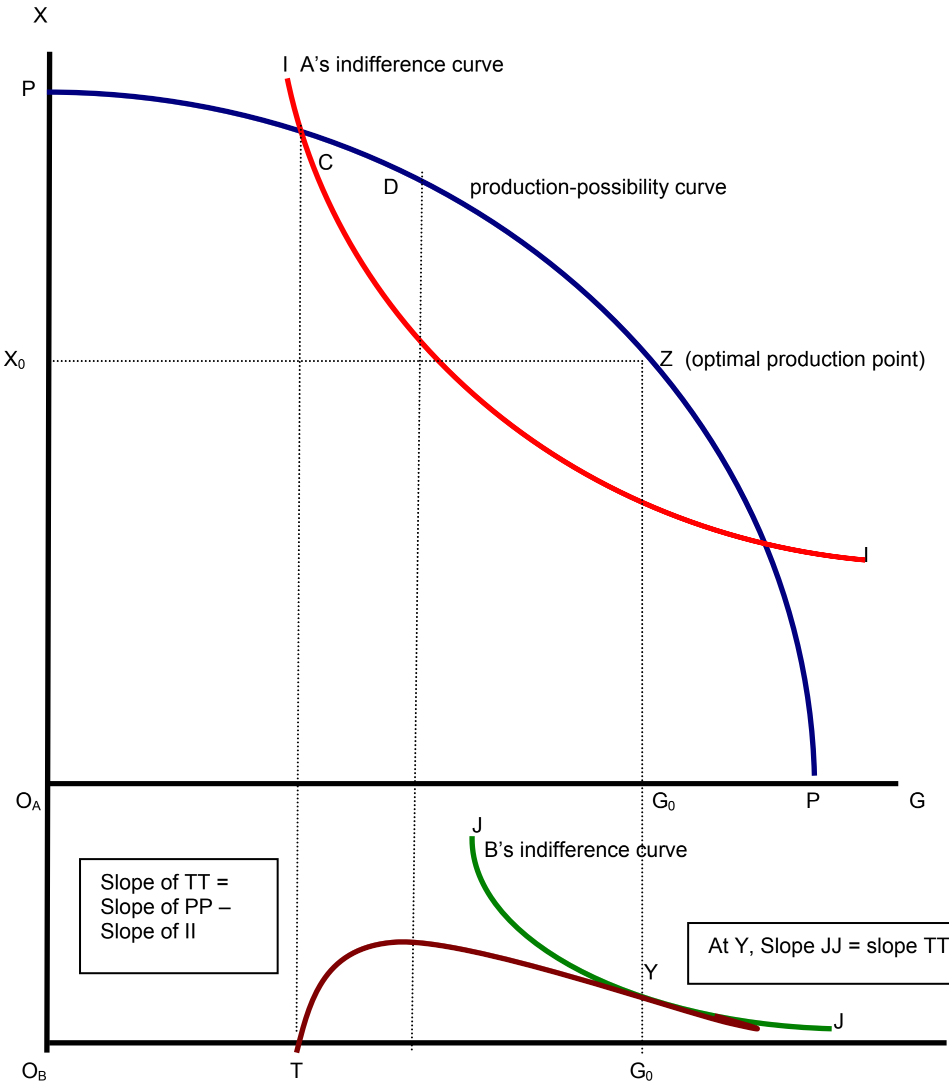

Figure 1:
The Samuelson Condition for the Efficient Provision of a Public Good

4. Free-Riding and the Private Provision of a Public Good

Suppose consumers 1 and 2 are independently deciding their contributions (G_1 and G_2) to the public good: $G_1+G_2=G$; $C(G)=G$. So if 1 thinks that 2 will contribute G_2, his problem is[5]:

$$\underset{G_1}{Max}\ U_1(W_1-G_1, G_1+G_2) \text{ s.t. } G_1 \geq 0 \tag{8}$$

The Kuhn-Tucker first-order conditions for solving this are:

$$\frac{\partial U_1(W_1-G_1, G_1+G_2)}{\partial G}-\frac{\partial U_1(W_1-G_1, G_1+G_2)}{\partial X_1} \leq 0 \tag{9}$$

which may be written as:

$$MRS_{XG}^{1}=\frac{\dfrac{\partial U_1(W_1-G_1, G_1+G_2)}{\partial G}}{\dfrac{\partial U_1(W_1-G_1, G_1+G_2)}{\partial X_1}} \leq 1 \tag{10}$$

where equality in equation (10) holds if $G_1>0$. So, if consumer 1 contributes a positive amount to the public good ($G_1>0$), he will equate his marginal rate of substitution between the private and public good to the marginal cost of providing the public good[6]. If $MRS_{XG}^{1}<1$, then he will not want to contribute ($G_1=0$) (Bergstrom, Blume and Varian, 1986).

In Figure 2, below, the initial endowment of consumer 1 is at point A (G_2, W_1) In left-hand panel he contributes an amount $G_1>0$ to the public good and moves to point B. In the right-hand panel, he 'free rides' on consumer 2's contribution of G_2 to the public good and remains at A ($G_1=0$).

Example: Suppose C(G)=G and the utility functions are (quasi-linear): $U_i(X_i, G)=X_i+\beta_i \log G$. Then the equilibrium conditions are: $(\beta_1/G)\leq 1$, $(\beta_2/G)\leq 1$. In general, only one of the constraints can be binding: if $\beta_2>\beta_1$, only consumer 2 will contribute and 1 will free ride. Only when $\beta_2=\beta_1$, will both contribute.

[5] Note that equation (8) incorporates the constraint: $X_1+G_1=W_1$

[6] Note that since, by assumption, C(G)=G, $MC_G=1$.

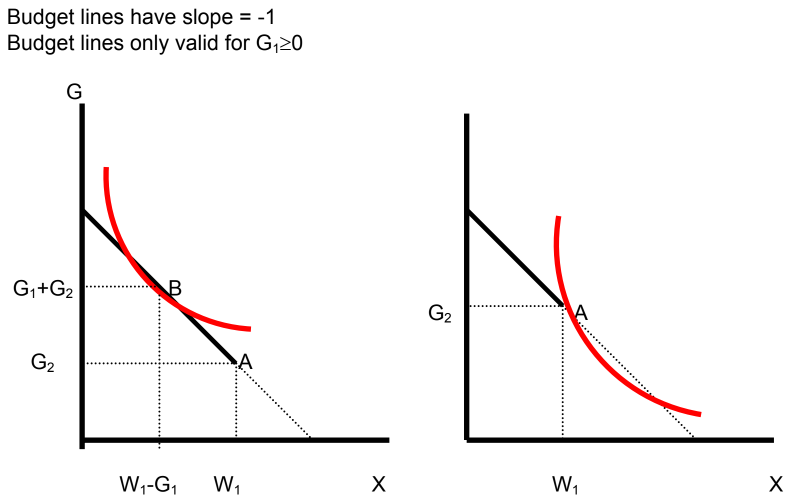

Figure 2:
Private Provision of a Public Good

Rewrite the consumer's optimisation problem of equation (8) as:

$$\underset{X_1,G}{Max}\ U_1(X_1,G) \text{ s.t. } G \geq G_2 \text{ and } G + X_1 = W_1 + G_2 \tag{11}$$

Solving this problem yields the consumer 1's demand function for the public good as: $G_1 = f_1(W_1 + G_2)$. Then the amount of the public good is:

$$G = Max\left\{f_1(W_1+G_2), G_2\right\} \Rightarrow G_1 = Max\left\{f_1(W_1+G_2) - G_2, 0\right\} \tag{12}$$

Equation (12), defines the reaction function for consumer 1 by giving his optimal contribution as a function of the contribution of consumer 2. A *Nash Equilibrium* is a set of contributions, G_1^*, G_2^* such that:

$$\begin{aligned} G_1^* &= Max\left\{f_1(W_1+G_2^*) - G_2^*, 0\right\} \\ G_2^* &= Max\left\{f_2(W_2+G_1^*) - G_1^*, 0\right\} \end{aligned} \tag{13}$$

5. Lindahl Pricing

Under *Lindahl pricing*, every consumer *i* is charged a price p_i for the public good and is offered the right to buy as much of the public good as he wishes at the price. Therefore, the maximisation problem for consumer *i* is:

$$Max\ U_i(X_i,G) \text{ s.t. } X_i + p_i G = W_i \tag{14}$$

and the first order condition for solving this problem is:

$$\frac{\partial U_i(X_i,G)/\partial G}{\partial U_i(X_i,G)/\partial X_i} = p_i \tag{15}$$

The optimal amount of the public good demanded by consumer *i* is: $G_i^* = G_i(W_i, p_i)$.

The question is: does there exist a set of prices $p_i^*, i = 1...N$, such that consumers will all choose an efficient amount of the public good: $G^* = G_1^* = G_2^* = .. = G_N^*$? By equation (7), an efficient amount of the public good must satisfy: $\sum_{i=1}^{N} \frac{\partial U_i(X_i^*, G^*)/\partial G}{\partial U_i(X_i^*, G^*)/\partial X_i} = C'(G^*)$ and so setting prices such that:

$$p_i^* = \frac{\partial U_i(X_i^*, G^*)/\partial G}{\partial U_i(X_i^*, G^*)/\partial X_i} \qquad (16)$$

will support an efficient amount of the public good. These prices - which are set equal to each consumer's marginal rate of substitution between the private and the public good – are known as *Lindahl prices* (Lindahl, 1919). These prices may also be interpreted as tax rates.

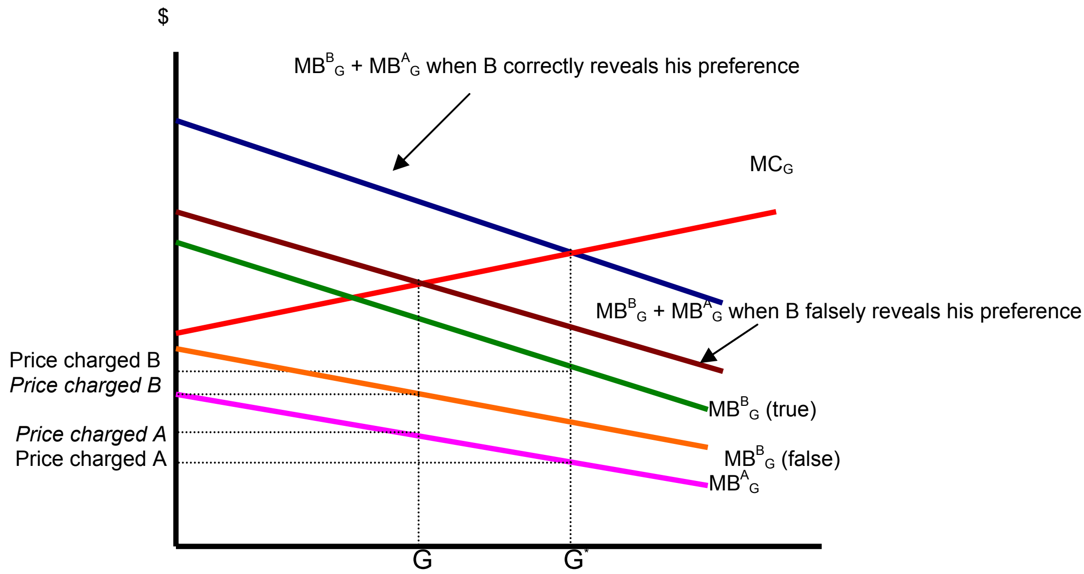

Figure 3:
Lindahl Prices and Under-Revelation of Preferences

In Figure 3, above, the optimal level of the public good is G*, when $MRS_{XG}^{1} + MRS_{XG}^{2} = MC_G$. At this level of provision, A and B pay, respectively, p_A and p_B, $p_A<p_B$. In order to avoid paying p_B, B under-declares his preference. As a consequence, provision of the public good falls to G, below the optimal level of provision, G*. So, Lindahl pricing could lead to under-provision of the public good if consumers falsify their preferences in order to reduce the prices (tax rates) they have to pay.

Lindahl pricing could also lead to under-provision of a public good if some consumers free ride on the demand of another consumer. In Figure 4, below, consumer A, free rides on B's demand for the public good: provision is at G_B, instead of at G^*, causing a net social loss equal to the area of the triangle XYZ.

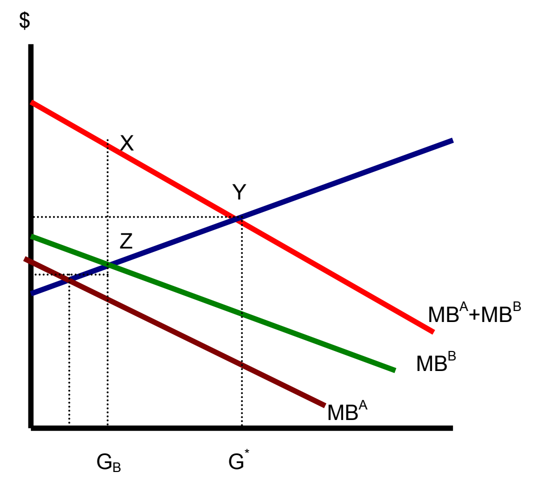

Figure 4:
Free riding Under Lindahl Pricing

6. Common Property Resources: Congestion but Non-Excludability

Consumption of a "common property" resource is rivalrous in the sense that each additional user of the resource reduces the return to the existing users. Hence there are social as well as private costs to adding new users. However, because all users (existing as well as potential) have free access to the resource, no one can be excluded from using the resource. Hence it is a "common property" or an "open access" resource.

Suppose that a community has access to a lake for fishing and that the lake has a limited number of fish which means that the more the number of fisherman that use the lake, the fewer the fish that each fisherman catches. This is shown in the table below.

Fishermen	*Fish per Fisherman*	*Total Catch*	*Marginal Catch*
1	10	10	10
2	9	18	8
3	8	24	6
4	7	28	4
5	6	30	2
6	5	30	0

Source: Liebowitz and Margolis (1999), p. 74

Suppose that the alternative wage for the fishermen is another occupation which pays (the equivalent of) six fish. Then, under open access, 5 fishermen will fish on the lake. But the *tragedy of the commons* is that the fourth and fifth fishermen will only generate, respectively, four and two additional fish. *From a social perspective*, their energies would have been better spent elsewhere, in the alternative occupation. But, since they only look to the average catch and not to the marginal catch, the lake is over-fished by two fishermen. This is depicted in Figure 5, below. The loss due to over-fishing is the area of the triangle, *ABC*.

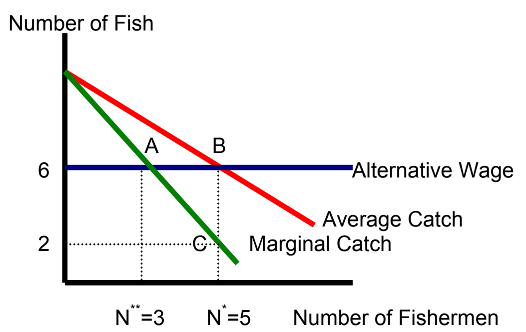

Figure 5:
The Tragedy of the Commons

Suppose it costs $*z* to 'equip' a fisherman. The total number of fish caught depends on how many fishermen are already there: let *f(N)* represent the number of fish caught, if there are *N* fishermen, so that *f(N)/N* is the average catch. To maximise the *social surplus* choose N so as to maximise:

$$\underset{N}{Max}\ f(N) - zN \tag{17}$$

and the first-order condition for this is:

$$f'(N) = z \tag{18}$$

Each potential fisherman, however, will compare the average catch, after he has joined, with the cost of fishing: if *f(N+1)/(N+1)>z* he will fish, otherwise he will not. But since *f(N+1)/(N+1)>f'(N)=f(N+1)-f(N)*, over-fishing will result[7].

7. Club Goods: Congestion and Excludability

There is a private good, with quantity denoted by *X* and a (club) public good, whose quantity is *G*. If S is the size of the membership of the club, then the representative member's utility function is:

$$U_i = U_i(X_i, G, S) \tag{1}$$

where: $\partial U_i / \partial X_i,\ \partial U_i / \partial G > 0$ and $\partial U_i / \partial S < 0$ if $S > \overline{S}$.

Each member tries to maximise (1) subject to a resource or budget constraint:

$$R_i = X_i + C(G,S)/S \tag{2}$$

where: *R* is the member's resources; X_i is the amount of the private good (with price set to unity) and *C(.)* is the club's cost of producing the club good.

The function C(.) is such that, for a given membership size S, cost increases with the level of provision (X); for a given provision level (*X*), for reasons of higher maintenance costs, provision costs also increase with the size of membership (*S*). The marginal costs of provision, with respect to level and membership, are both positive:

$$\frac{\partial C}{\partial G} > 0, \frac{\partial C}{\partial S} > 0$$

Differentiating the Lagrangean function:

$$L = U_i(X_i, G, S) + \lambda[Y_i - X_i + C(G,S)/S]$$

with respect to X_i, *G* and *S* yields the first-order conditions:

$$\begin{aligned} MRS_{XG} &= \frac{\partial U_i / \partial G}{\partial U_i / \partial X_i} = \frac{\partial C}{\partial G} \times \frac{1}{S} \quad \text{(provision)} \\ MRS_{XS} &= \frac{\partial U_i / \partial S}{\partial U_i / \partial X_i} = \frac{S(\partial C / \partial S) - C(G,S)}{S^2} = \frac{1}{S}\frac{\partial C}{\partial S} - \frac{C(G,S)}{S^2} \quad \text{(membership)} \end{aligned} \tag{3}$$

The provision condition says that the MRS between the private and the club good is equated to the individual's share in the marginal cost of provision with respect to the level of the good. The membership condition says that the MRS between the private good and membership size is equated to the marginal cost of increasing membership

[7] When the average is falling, the marginal lies below the average: $d(f(N)/N)/dN < 0 \Rightarrow df(N)/dN < f(N)/N$

size: the first component of this is positive ($\frac{1}{S}\frac{\partial C}{\partial S}$>0) since increased membership increases provision cost; the second component of this is negative (-$\frac{C(G,S)}{S^2} < 0$) since increased membership reduces membership fees (i.e. the individual's share of costs). Cross-multiplying the provision condition in equation (3) by S yields Samuelson's (1954) condition for the optimal provision of public good:

$$\sum MRS_{XG} = MC_G$$

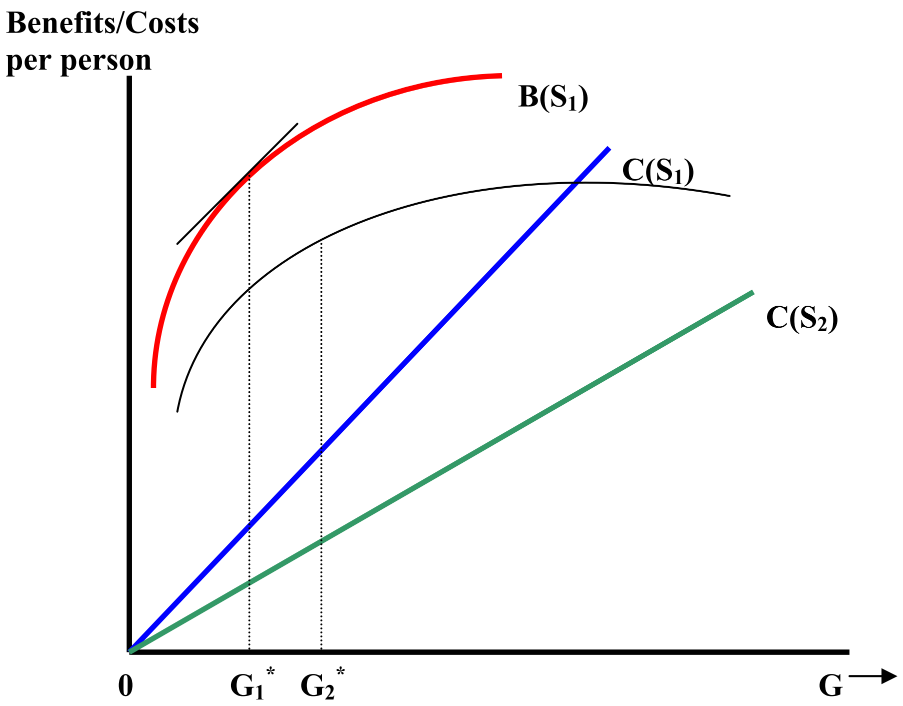

Figure 1
Optimal provision level for a club good

Source: Cornes and Sandler(1996), p. 361

In Figure 1, above, it is assumed that the club good is produced under constant returns to scale - the average cost curves, $C(S_1)$ and $C(S_2)$, corresponding to different levels of membership, $S_1 < S_2$, are linear. The curves, $B(S_1)$ and $B(S_2)$, are the corresponding benefit levels: these are concave, showing marginal utility diminishes as the level of provision increases. For a membership level, S_1, the optimal provision is G_1^*, at which the gap between the benefit and cost curves, $B(S_1)$ and $C(S_1)$, is maximum.

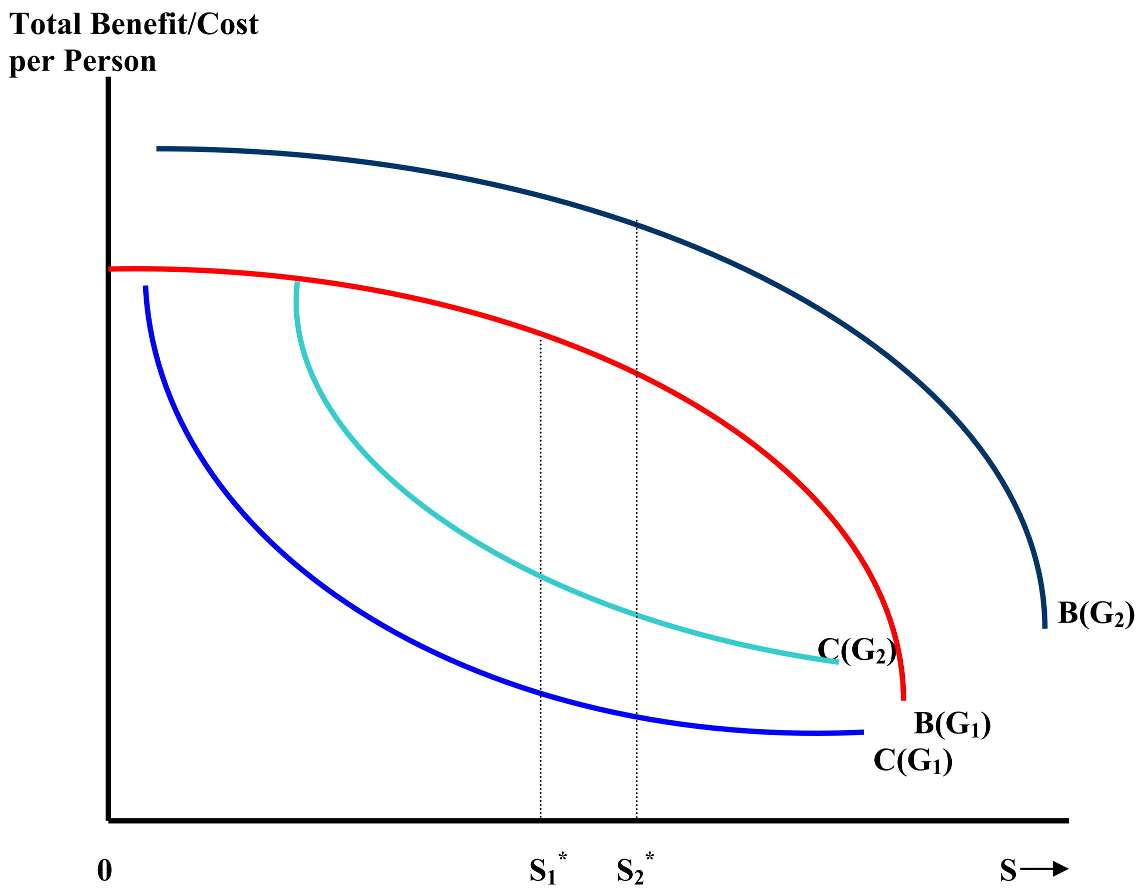

Figure 2
Optimal Level of Membership

In Figure 2, the benefit curve, $B(G_1)$, shows the benefit level per member from a level of provision of G_1 of the club good, for varying levels of membership. The curve slopes downward showing, due to congestion costs, the benefit from a fixed amount of the club good falls as membership increases; $B(G_2)$ shows the benefit levels from a higher level of provision, G_2, of the club good - it lies above $B(G_1)$. The cost curves,

$C(G_1)$ and $C(G_2)$, show the cost per member from levels of provision of G_1 and G_2, respectively, of the club good, for varying levels of membership: since costs are equally shared, C(.) is a rectangular hyperbola. The optimal levels of membership (the levels which maximise the distance between the cost and the benefit curves) are S_1^* and S_2^* for levels of provision of G_1 and G_2.

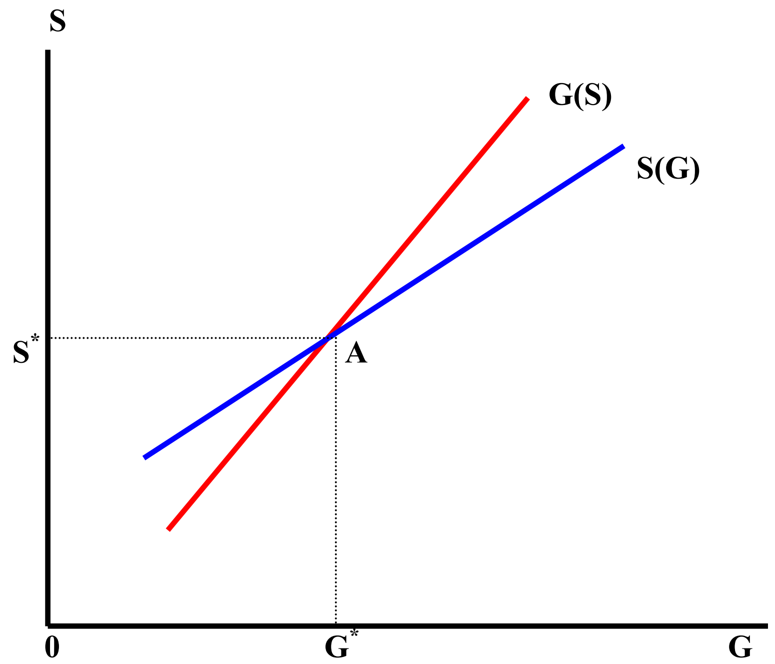

Figure 3
Equilibrium Level of Provision and Membership

In Figure 3, the G(S) curve (from Figure 1) shows that the optimal provision of the club good rises as membership is increased. The S(G) curve (from Figure 2) shows that the optimal size of membership is higher for higher levels of provision. The equilibrium level of provision and membership is given at the point of intersection of the two curves. For a *stable* equilibrium, the S(G) curve should be flatter than the G(S) curve. The optimal number of clubs, K^* is obtained by dividing the size of the population (P) by the size of the optimal membership (S^*)

Community Size and the Tiebout hypothesis

Suppose that the clubs represent local communities each supplying the "club good" (a "package" of education, health, transport etc. services) to the local population. For each community there is an optimal size S^* which will maximise the net benefit from the package, $\bar{G}$: $B(\bar{G},S) - C(\bar{G}, S)$. If $S>S^*$, people will leave the community and if $S<S^*$, people will enter the community until each community is of optimal size. This is Tiebout's (1956) model of "voting with the feet" between communities offering a package of local services.

8. Variable-Use Public Goods

Suppose there are N cars using a bridge of capacity K with an average crossing time of $T(N,K)$: the total time in crossing the bridge is $N \times T(N,K)$. An additional car increases congestion on the bridge and the average crossing time increases to $dT(N,K)/dN$. So, the total time in crossing the bridge increases by $N \times (dT(N,K)/dN)$. This increase is the *social marginal cost* imposed by the additional traveller.

In Figure 7, below, the optimal number of cars crossing the bridge is N^{**} but actually N^* cars use the bridge (at N_0, there are no congestion costs). This is because new users do not take account of the additional cost they impose on all the existing users. Charging a toll on the bridge removes the excess usage.

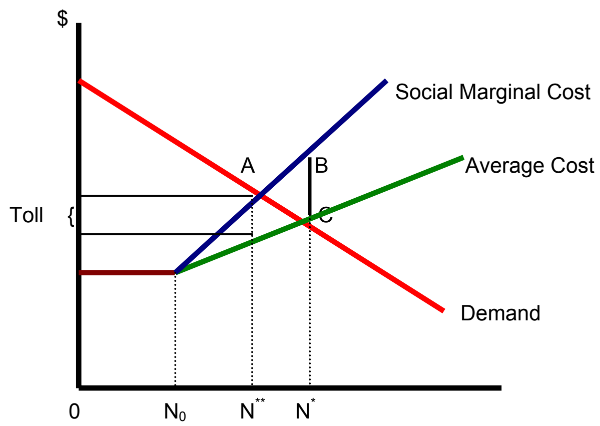

Figure 7
Congestion Costs

8. The Impossibility Theorem and Cyclical Voting

Given that we are able to identify the conditions for a Pareto efficient allocation between public and private goods (see equation (7), above), the question rises about the mechanisms available for achieving this allocation. In the case where all goods are private goods, the second Fundamental Theorem of Welfare Economics informs us that the price mechanism provides a means of attaining a Pareto efficient allocation between the different types of goods. However, because of the free-rider problem, private provision of public goods is not feasible.

In democracies, the voting mechanism offers consumers a means of deciding on the level of provision of public goods. However, voting as means of deciding on public good provision runs into the problems first noted by Arrow (1951) in his *Impossibility Theorem* in which he showed that any social rule which satisfied a minimal set of fairness conditions could produce an intransitive ranking when two or more persons had to choose from three or more projects. These conditions were the axioms of: *unrestricted domain* (individuals had transitive preferences over all the policy alternatives); *Pareto choice* (if one project made someone better off than another project, without making anyone worse off, then it would be the socially preferred choice); *independence* (the ranking of two choices should not depend on what the other choices were); *non-dictatorship* (the social ordering should not be imposed).

The voting problem is one of selecting, on the basis of the declared preferences of the electorate, one out of an available set of options. Stated in this manner, the voting problem is akin to the problem of social choice where individual preferences are used in order to arrive at a notion of 'social welfare'.

Every individual in society may rank different 'projects' according to the net benefits that they expect to obtain.

Table 1: Condorcet Winner

	23 voters	*19 voters*	*16 voters*	*2 voters*
1st preference	A	B	C	C
2nd preference	C	C	B	A
3rd preference	B	A	A	B

Condorcet in 1785 suggested a pair-wise comparison of alternatives, choosing, at each comparison, the alternative with greater support. An alternative that wins over all the others is then selected as the preferred option and is termed the *Condorcet winner*. Thus, in Table 1, the Condorcet winner C beats A, 37-23 and beats B, 41-19. However, as Table 2 shows, a Condorcet winner need not exist: Table 2 demonstrates the phenomenon of 'cyclical voting' - also termed the 'paradox of voting' - whereby A beats B (33-27); B beats C (42-18); and C beats A (35-25).

Table 2: The Paradox of Voting

	23 voters	*17 voters*	*2 voters*	*10 voters*	*8 voters*
1st preference	A	B	B	C	C
2nd preference	B	C	A	A	B
3rd preference	C	A	C	B	A

'Cyclical voting' is the political illustration of The Impossibility Theorem: a ranking by individuals may not lead to a social ranking, that is to a ranking to which all individuals in society would subscribe. For example, with three individuals (A, B and C) and three projects (X, Y and Z) suppose the rankings are as given in the table below:

Table 3: Cyclical Social Preference under Pair-wise Voting

Preference Ordering	*A*	*B*	*C*
First Choice	**X**	**Z**	**Y**
Second Choice	**Y**	**X**	**Z**
Third Choice	**Z**	**Y**	**X**

Then in a sequence of pair-wise comparisons: X versus Y, Y wins since both A and B prefer X to Y; Y versus Z, Y wins, since both A and C prefer Y to Z; X versus Z, Z wins since both B and C prefer Z to X. The implied social ordering is that X is preferred to Y; Y is preferred to Z; but Z is preferred to X! The cyclical nature of social preferences arises from the fact that the social ordering is not transitive or, in the language of electoral studies, there is no *Condorcet winner*. Indeed, the problem of social choice is not unlike that of voting behaviour: in both cases the issue is one of translating individual preferences into an agenda for collective action that faithfully represents these preferences. This was a point noted by Black (1948).

Single- and Multi-Peaked Preferences

The question, therefore, is whether it is possible to specify conditions under which cyclical voting will not occur. This was addressed by Black (1948) using the concept of 'single-peaked' preferences. Suppose that the set of alternatives can be represented in one dimension - for example, choice between different levels of public expenditure - and suppose that for each voter there is a preferred level of expenditure - which may be different for different voters - such that preferences drop monotonically for levels on either side of this optimum. In such a case (see Figure 4) voter preferences are said to be *single-peaked*. This means that the greater the distance of the actual position from the unique utility maximising position, the lower the level of utility.

Under single-peaked preferences the median voter decides in the sense that the preferred choice of the median voter is the Condorcet winner. This result is illustrated in Figure 8 (taken from Mueller, 2003) in which there are five voters – voters 1 to 5- each with single-peaked preferences. In a pair-wise contest, the preferred choice of the median voter, Voter 3, will beat the preferred choice of all other voters.

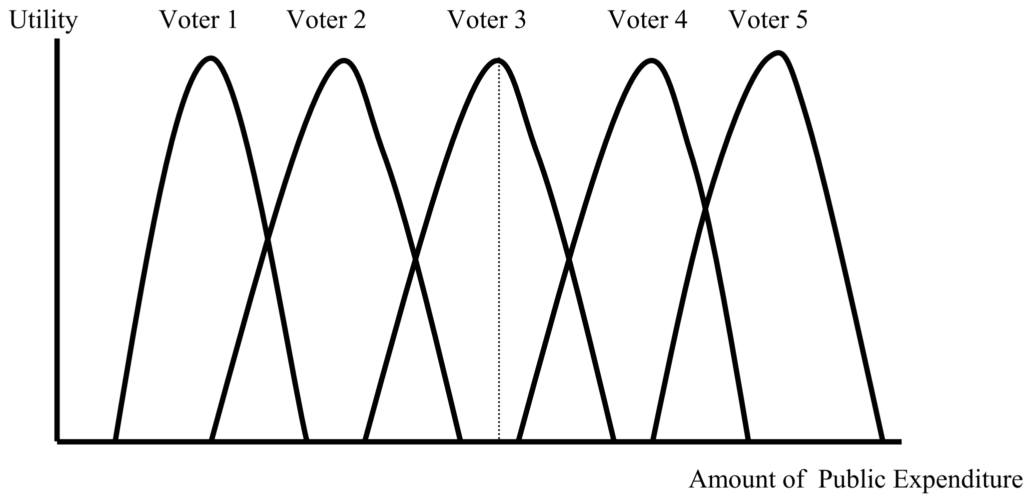

Figure 8:
The Median Voter Decides

However, when the options before the voters concern the *type* of expenditure, rather than the *amount* of expenditure, multiple peaked expenditures cannot be ruled out. For example (Connolly and Munro, 1999), suppose three parties are trying to decide on the best way to spend £100 million. The options are: buy a nuclear submarine for the Navy; spend it on higher education; embark on a major programme of improved roads. The three parties – the Conservatives, the Liberal Democrats and Labour – set out their preferences as follows:

Table 4: Multi-Peaked Preferences

Party→ *Ranking↓*	**Conservatives**	**Lib Dems**	**Labour**
1	Submarine	Education	Roads
2	Education	Roads	Submarine
3	Roads	Submarine	Education

Now the Labour Party exhibits multi-peaked preferences, while both the Conservatives and the Liberal Democrats have single-peaked preferences (see Figure 9). The consequence of multi-peaked preferences is that in binary comparisons: submarine beats education (Conservatives + Labour against the Lib Dems); education beats roads (Conservatives + Lib Dems against Labour); but roads beats submarine (Labour + Lib Dems against the Conservatives).

A very important lesson from multi-peaked preferences is that the outcome depends very much on the *order* in which the options are voted for. So, if the first vote was education versus roads, education would win; if the next vote was between submarine versus education, the submarine would win and £100 million would be spent on the submarine. However, if the first vote was submarine versus roads, roads would win; if the next vote was roads versus education, education would win and £100 million would be spent on education. So astute chairmanship of meetings is important to ensure the "desired" outcome!

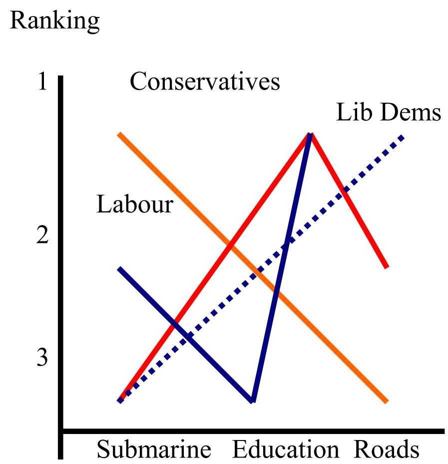

Figure 9:
Multi- and Single-Peaked Preferences

However, if instead of as in Table 4, preferences were represented by Table 5, then the preferences of the Labour party would also be single-peaked: this is illustrated by the dotted line in Figure 5. Under single-peaked preferences: education is preferred to roads (Conservatives + Lib Dems versus Labour); roads are preferred to the submarine (Lib Dems + Labour versus the Conservatives); and education is preferred to the submarine (Labour + Lib Dems versus the Conservatives). Preferences are, therefore, transitive: the £100 million is spent on education.

Table 5: Single-Peaked Preferences

Party→ *Ranking↓*	Conservatives	Liberal Democrats	Labour
1	Submarine	Education	Roads
2	Education	Roads	Education
3	Roads	Submarine	Submarine

Desirable Voting Systems

The relevance of the work of Black (1948) and of Arrow (1951) to the voting problem lay in attempting to identify: (a) the desirable conditions that any voting system should satisfy and (b) a voting system that satisfied these conditions. May (1952) showed that when there were only two alternative candidates or parties, majority rule (i.e. the candidate with the majority of votes being elected) was unambiguously the best. The problem was to extend this result when there were more than two alternatives. In such situations, with more than two candidates or parties, different voting systems could be constructed, all of which seemed fair and reasonable - and all of which, in the event of two alternatives, yielded majority rule - but which, nevertheless, yielded different outcomes.

One possible system is plurality ('first-past-the-post') in which each voter votes for exactly one candidate and the candidate receiving the largest number of votes wins, which is the system that applies to elections to Westminster. A problem with this system is that it is based on an incomplete revelation of preferences: there is no requirement for a voter to rank the options for which he (she) did not vote. As Table 2, above, shows, on the basis of votes cast by 60 voters, A wins by plurality, yet A would lose against B alone (25 to 35) and against C alone (23 to 37).

This then points to a second defect of plurality voting which is the fact that it is subject to agenda manipulation and that the presence, or absence, of options - even if those options cannot win - can affect the outcome. In the Table 2, if either B or C was "persuaded" not to stand, the other would win.

The alternative is for each voter to rank the alternatives in order of preference (as in Table 2 above) and then the appropriate electoral rule would aggregate these individual rankings into an overall ranking. Such a procedure is termed an 'ordinal procedure'. One possible electoral rule, based on an ordinal procedure, is the *Borda count*: in the presence of N options, assign N points to the option ranked first, N-1 points to the option ranked second and finally one point to the option ranked last. A Borda count applied to the data in Table 2 sees C a comfortable winner with 138 points, A coming second with 108 points and B finishing last with 114 points. The Borda count method, however, is also susceptible to false revelation of preferences: voters, irrespective of their true preferences, would be inclined to give the lowest preference vote to the candidate they thought was most threatening to their preferred candidates electoral prospects.

Both plurality and ordinal procedures may be multistage procedures - so that the chosen option only emerges after successive rounds of voting - by combining either of them with the possibility of elimination. Thus, plurality plus run-off eliminates all but the two strongest candidates in the earlier rounds leaving a simple run-off between the two candidates for the final round. An alternative is to eliminate in each round the weakest candidate and to choose a candidate after N-1 rounds of voting. Although both these voting procedures - and variants thereof - are reasonable they don't necessarily lead to the same outcome. For example, in Table 6, taken from Miller (1987): C wins under plurality; A, with 50 points, wins under a Borda count; and B wins against C either under plurality with run-off or with successive elimination of the weakest candidate.

Table 6: Multi-Stage Voting

	4 voters	*4 voters*	*2 voters*	*9 voters*
1st preference	A	B	B	C
2nd preference	B	A	D	D
3rd preference	D	D	A	A
4th preference	C	C	C	B

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Mathematical Appendix

Deriving The Samuelson Condition for the Efficient Allocation of a Public Good

$\underset{X_1, X_2, G}{Max} \; U_1(X_1, G)$ s.t. $U_2(X_2, G) = \bar{U}_2$ and $X_1 + X_2 + C(G) = W_1 + W_2$ (4)

Form the Lagrangian:

$$L = U_1(X_1, G) - \lambda[U_2(X_2, G) - \bar{U}_2] - \mu[X_1 + X_2 + C(G) - (W_1 + W_2)] \quad (5)$$

and differentiate with respect to X_1, X_2, and G to obtain the first-order conditions as:

$$\frac{\partial L}{\partial X_1} = \frac{\partial U_1(X_1, G)}{\partial X_1} - \mu = 0$$

$$\frac{\partial L}{\partial X_2} = -\lambda \frac{\partial U_2(X_2, G)}{\partial X_2} - \mu = 0 \quad (6)$$

$$\frac{\partial L}{\partial G} = \frac{\partial U_1(X_1, G)}{\partial G} - \lambda \frac{\partial U_2(X_2, G)}{\partial G} - \mu C'(G) = 0$$

From the first equation: $\mu = \frac{\partial U_1(X_1, G)}{\partial X_1}$;

from the second equation: $\frac{\mu}{\lambda} = -\frac{\partial U_2(X_2, G)}{\partial X_2}$;

and from the third equation: $\frac{1}{\mu}\frac{\partial U_1(X_1, G)}{\partial G} - \frac{\lambda}{\mu}\frac{\partial U_2(X_2, G)}{\partial G} = C'(G)$

Substituting the appropriate expressions for μ and (μ/λ) into the third equation yields the equilibrium condition:

$$\frac{\partial U_1(X_1, G)/\partial G}{\partial U_1(X_1, G)/\partial X_1} + \frac{\partial U_2(X_2, G)/\partial G}{\partial U_2(X_2, G)/\partial X_2} = C'(G) \quad (7)$$

or, in other words:

$$MRS^1_{XG} + MRS^2_{XG} = MC_G$$

A Nash Equilibrium Interpretation of Common Property Resources

There are N farmers in a village who graze their cows on the village green. This is owned in common by all the villagers. The number of goats owned by the ith farmer is g_i and $G = \sum_i g_i$ is the total number of goats grazing on the green. The price of a goat is *c* and *v(G)* is the value of the milk from a goat when there are *G* goats grazing. The maximum number of goats that can be grazed on the green is $\bar{G}$: *v(G)=0* if G=$\bar{G}$, while *v(G)>0* if G< $\bar{G}$. The 'strategy' for each farmer is to choose his g_i from his 'strategy set': *[0,∝]* . The payoff to the farmer from g_i goats depends upon his choice, as well as upon the choices made by others:

$$\pi_i = g_i v(g_1 + .. + g^*_{i-1} + g_i + g^*_{i+1} + ... + g_N) - cg_i \quad (8)$$

If $g_1^* ... g_N^*$ is to be a Nash-equilibrium, then, for each farmer *i, i=1...N,* g_i^* should maximise π_i, given that the other farmers choose $g_j^*, j = 1...N, j \neq i$. The first-order conditions for maximising π_i wrt g_i are:

$$v(g_i + \mathbf{g_j^*}) + g_i v'(g_i + \mathbf{g_j^*}) - c = 0 \qquad (9)$$

where: $\mathbf{g_j^*} = \sum_{\substack{j=1 \\ j \neq i}}^{N} g_j^*$ and, for a Nash equilibrium, g_i^* solves (24). Consequently, the conditions for a Nash equilibrium are:

$$v(G^*) + g_i^* v'(G^*) - c = 0,\ i = 1...N \qquad (10)$$

where: $G^* = \sum_{i=1}^{N} g_i^*$, $v'(G^*) = \dfrac{\partial v(G^*)}{\partial G^*}$

Interpretation: There are G^* goats being grazed, so payoff per goat is $v(G^*)$. A farmer is contemplating adding a goat. This goat will give a payoff of $v(G^*)$ but it will reduce the payoff from his existing goats by the reduction in the payoff-per-goat (after another goat has been added), $v'(G^*)$, times the number of goats he owns, g_i^*. This is his *marginal private benefit* from adding another goat. He compares this marginal private benefit ($v(G^*) + g_i^* v'(G^*)$) to the cost of a goat, *c*, and decides accordingly. Note that when $g_i^* = 1$ (he owns only one goat), $v(G^*) + g_i^* v'(G^*)$ is the new average payoff from a goat.

Summing over the farmers' first-order conditions and dividing by N, yields:

$$v(G^*) + (G^* / N) v'(G^*) - c = 0 \qquad (11)$$

and solving (26) yields the Nash equilibrium (total) number of goats, G^*.

In contrast, the social optimum number of goats, G^{**} is given by maximising the net revenue to the village from the goats:

$$Gv(G) - Gc \qquad (12)$$

and this yields as first-order conditions:

$$v(G^{**}) + G^{**} v'(G^{**}) - c = 0 \qquad (13)$$

Interpretation: There are G^{**} goats being grazed, so payoff per goat is $v(G^{**})$. The village is contemplating adding a goat. This goat will give a payoff of $v(G^{**})$ but it will reduce the payoff from the existing goats in the village by the reduction in the payoff-per-goat (after another goat has been added), $v'(G^{**})$, times the number of goats in the village, G^{**}. This is the *marginal social benefit* from adding another goat. The village compares this marginal private benefit ($v(G^{**}) + G^{**} v'(G^{**})$) to the cost of a goat, *c*, and decides accordingly. Comparing (26) with (28) shows that $G^* > G^{**}$. The common resource is over utilised because each villager considers the effect of his action (of grazing another goat) on only his own welfare and neglects the effect of his action upon the other villagers.