Week 6
Market Failure due to Externalities

1. Externalities

An *externality* exists when the action of one agent unavoidably affects the welfare of another agent. The affected agent may be a consumer, giving rise to a *consumption externality*, or a producer, giving rise to a *production externality*.

Suppose there are two firms. Firm 1 produces output y at a cost $C(y)$ and imposes an externality in the form of a cost $E(y)$ on firm 2. If p is the price of firm 1's output, profits for the firms are:

$$\pi_1 = py - C(y) \text{ and } \pi_2 = -E(y) \qquad (1)$$

Firm 1 chooses output by maximising π_1 and the profit maximising level of output occurs when: $p = C'(y)$. However, since Firm 1 takes account only of private costs, the profit-maximising output, y^* is, from a social perspective, too high. If the two firms merged in order to *internalise* the externality, the merged firm would maximise:

$$\pi = py - C(y) - E(y) \qquad (2)$$

and the profit maximising output would occur when: $p = C'(y) + E'(y)$ that is when price was equal to marginal social cost. The solution to (2) would yield the social optimum, y^{**}. In Figure 1, below, the social cost of the externality is the area of the triangle, *ABC*.

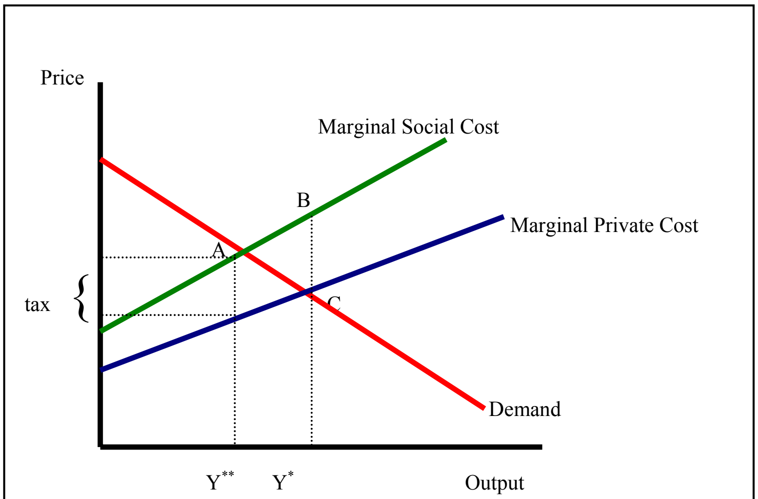

Figure 1:
The Social Cost of An Externality

2. Modelling Externalities

There are two consumers (*i=1,2*) and *N* commodities *(r=1…N)*. The utility of consumer *i* depends upon the quantities consumed by him/her of the commodities ($X_r^i,\ r=1,..,N$) *and upon the level, denoted* A^j*, of some action taken by the other consumer :*

$$U^i = U^i(X_1^i, X_2^i, ..., X_N^i; A^j) \qquad (3)$$

It is assumed that:

(i) consumer *j* can take the action, for any level A^j, *without any monetary cost*.

(ii) The action taken by *j* imposes an *externality* upon *i*, if $\frac{\partial U^i}{\partial A^j} \neq 0$. The externality is *positive* if $\frac{\partial U^i}{\partial A^j} > 0$ and *negative* if $\frac{\partial U^i}{\partial A^j} < 0$

If $V^i(p, W^i, A^j)$ represents *i's indirect utility function*, then:

$$V^i(\mathbf{p}, W^i, A^j) = \underset{X_r^i}{Max}\ U^i(X_1^i, ..., X_1^i; A^j) \text{ st } \sum_{r=1}^{N} p_r X_r^i = W^i \qquad (4)$$

where: W^i is the wealth of the consumer, p_r is the price of the commodity *r* and $\mathbf{p} = \{p_r\}$ is the price vector.

It is assumed that the utility function, U^i is *quasi-linear* with respect to some *numeraire* commodity: $U^i(X_1^i, .., X_N^i; A^j) = X_1^i + G^i(X_2^i, .., X_N^i; A^j)$. Then, holding prices constant, the indirect utility function takes the form:

$$V^i(W^i, A^j) = \phi^i(A^j) \qquad (5)$$

Consumer *j* will choose A^j so as to maximise $\phi^j(A^j)$ and the first order condition for this is:

$$\phi^{j\prime}(A^{j*}) = 0 \qquad (6)$$

The action level A^* (dropping the superscript *j*) represents *j's* optimal action and the combined utilities at his private optimum is:

$$\phi^i(A^*) + \phi^j(A^*) = 0 \qquad (7)$$

where: A^* satisfies condition (6).

The socially optimal action, however, would be for *j* to choose A^j so as to maximise:

$$\phi^i(A^j) + \phi^j(A^j) \qquad (8)$$

and the first order condition for this is:

$$\phi^{i\prime}(A^j)+\phi^{j\prime}(A^j) \tag{9}$$

Condition (9) yields the *socially optimal* action by j, A^{**} in contrast to his/her *privately optimal* action, A^*. If $\phi^i(A^j)<0$, so that the externality is negative, by conditions (6) and (9): $\phi^j(A^*)=0$, but $\phi^j(A^{**})>0 \Rightarrow A^{**}$<A* . By contrast, if $\phi^i(A^j)>0$, so that the externality is positive, by conditions (6) and (9): $\phi^j(A^*)=0$, but $\phi^j(A^{**})<0 \Rightarrow A^{**}$>A* .

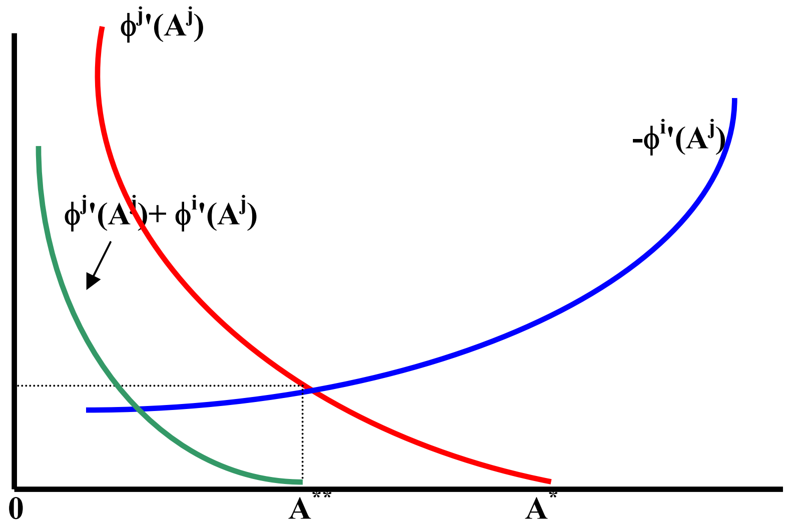

Figure 1
A Negative Externality

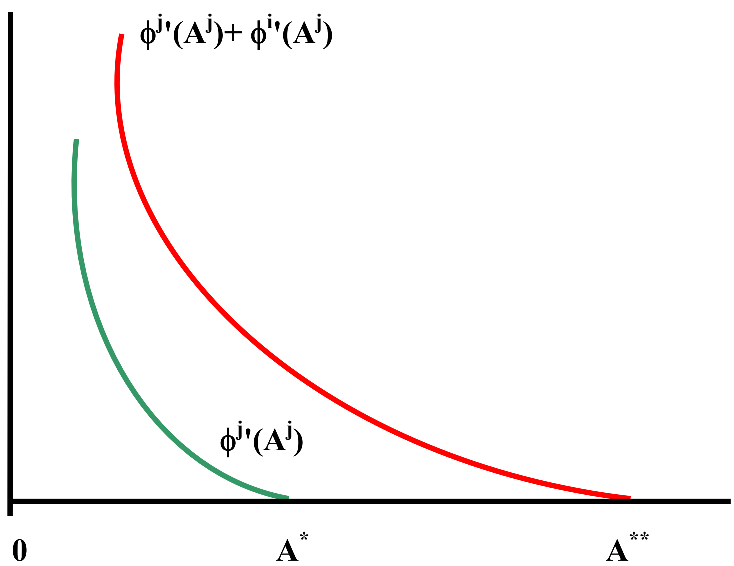

Figure 1
A Positive Externality

3. Solutions to Externalities: Taxes and Subsidies

The externality problem arises because consumer *j* - who is taking the externality-producing action - faces the 'wrong' prices and these lead it to take action at level A^* instead of at A^{**}. It is assumed that the externality is negative so that $A^{**} < A^*$. A corrective tax (first proposed by Pigou, 1920) can be imposed which, by yielding the 'right' prices, will induce it to act at the level A^{**}. If t is the tax rate per unit of action taken, then consumer *j's* optimisation problem would be:

$$\underset{A^j}{Max}\ \phi^j(A^j) - tA^j \tag{10}$$

and the first order condition for this is: $\phi^{j\prime}(A^*) = t$. If the tax rate, *t*, is set at $t = \phi^{i\prime}(A^{**})$ (i.e. to consumer *i's* marginal damage when consumer *j* takes action $A^j = A^{**}$), then $A^* = A^{**}$. The problem with this solution is that it requires the taxing authority to know the externality damage function $\phi^{i\prime}(A^j)$ and, hence, the socially optimal action level , A^{**}. But, if this was known, consumer *j* could simply be regulated to produce A^{**}.

Under a subsidy, consumer *j* is paid an amount s per unit of action to reduce his/her externality-producing action, A^j. Now consumer *j's* objective function is:

$$\phi^j(A^j) + (A^* - A^j) \times s \tag{11}$$

and the first order condition for solving this problem is: $\phi^{j\prime}(A^j) = s$. If the subsidy rate, *\$s*, is set at $s = \phi^{i\prime}(A^{**})$ (i.e. to consumer *i's* marginal damage when consumer *j* takes action $A^j = A^{**}$), then $A^* = A^{**}$.

The difference between a tax and a subsidy is that, under a tax, consumer *j* makes payment while, under a subsidy, he/she receives payment. Under a subsidy, consumer *j*, by reducing his/her action by one unit, forgoes $\phi^{j\prime}(A^j)$ of utility but gains *\$s* by way of subsidy.

4. Missing Markets

The reason that the 'externality problem' arises is that there is no market for the externality-producing action. Consumer *j* undertakes the externality-producing action, up to any level that he/she chooses, *without any cost*. The action is, therefore, a free good because there does not exist a market in which it is traded. The problem is, therefore, one of *missing markets* and *ipso facto* may be solved by creating a market for the externality-producing action.

Suppose that consumer i has the right to be free of the externality but can sell to consumer j the right to undertake the externality-producing action at a price of $\$q$ per unit. Then the optimising problem facing consumer i is to decide how many units of the action to allow j to undertake. This is achieved by solving:

$$\underset{A^j}{Max}\ \phi^i(A^j) + qA^j \tag{12}$$

The first order condition for solving (12) is:

$$-\phi^{i\prime}(A^j) = q \tag{13}$$

which implicitly defines a supply function: $A_S = S(q)$.

Consumer j has to decide how many units of the action to buy and he/she does so by maximising:

$$\phi^j(A^j) - qA^j \tag{14}$$

The first order condition for solving (14) is:

$$\phi^{j\prime}(A^j) = q \tag{15}$$

which implicitly defines a demand function: $A_D = D(q)$.

The equilibrium price, q^*, clears the market so that:

$$D(q^*) = S(q^*) \tag{16}$$

and at the equilibrium price, by equations (13) and (15):

$$\phi^{i\prime}(A^j) = -\phi^{j\prime}(A^j) \tag{17}$$

which is the condition for a social optimum.

5. Property Rights

This solution is due to Coase (1960) and is often referred to as the 'Coase Theorem'. This theorem says that when parties can bargain to their mutual advantage *without cost*, then the resulting outcome will be efficient, *regardless of how property rights are distributed*. Put differently, the 'Coasian solution' to the problem of externalities is to establish institutions which will define and enforce property rights and which will allow parties to bargain at zero transaction cost.

Coase's first point was that externalities are the joint product of the 'offender' and the 'victim' *and the most efficient system of avoiding an externality is to put the onus for avoidance on the party which can avoid it at the least-cost*. The Pigovian solution of penalising the generator of externalities would only be efficient if this party was the lowest cost avoider; otherwise, it would be inefficient.

Coase's second point was that in order to remove the ill-effect of an externality, neither regulation nor taxes were necessary. If transaction costs were zero – that is, any agreement that is in the mutual interest of the parties can be arrived at costlessly – then bargaining between the parties would lead to an efficient outcome, regardless of how property rights were defined.

Coase's third point was that the problem was not one of externalities but, rather, one of transaction costs which prevented externalities being bargained out of existence. When we observed externalities in the real world, Coase would have us enquire about the level of transaction costs which prevented externalities being bargained away.

Suppose consumer i has the right to be free of the externality. Then consumer i can sell to consumer j the right to undertake the externality producing action, at a level A^j, in exchange for a payment, T^j. If consumer j rejects this offer, then he/she cannot take the externality producing action and $A^j = 0$. Then consumer j will accept this offer if, and only if:

$$\phi^j(A^j) - T^j \geq \phi^j(0) \tag{18}$$

Given constraint (18), consumer i will choose (A^j, T^j) so as to maximise:

$$\phi^i(A^j) + T^j \text{ st } \phi^j(A^j) - T^j \geq \phi^j(0) \tag{19}$$

Since consumer i prefers a higher, to a lower, T^j, the constraint will bind at the optimum and the optimising problem of (18) becomes:

$$\underset{A^j}{Max}\ \phi^i(A^j) + \phi^j(A^j) - \phi^j(0) \tag{20}$$

and the first order conditions for solving this are:

$$\phi^{i\prime}(A^j) = \phi^{j\prime}(A^j) \tag{21}$$

which is the condition for a social optimum. Then consumer i chooses $A^j = A^{**}$ and $\hat{T} = \phi^j(A^{**}) - \phi^j(0)$. This offer ($A^{**}, \hat{T}$) is accepted by consumer j.

Suppose consumer j has the right to undertake the externality producing action up to any desired level. Consumer i can offer consumer j the level of action A^j in exchange for a payment, from i to j, of T^i. If consumer j rejects this offer, consumer *j* has the right to undertake the action to any desired level, A^*, where this will satisfy: $\phi^{j\prime}(A^*) = 0$. So consumer *j* will accept this offer if, and only if:

$$\phi^j(A^j) + T^i \geq \phi^j(A^*) \tag{22}$$

Given constraint (22), consumer *i* will choose A^j and T^i so as to:

$$\underset{A^j, T^i}{Min}\ \phi^i(A^j) + T^i \text{ st } \phi^j(A^j) + T^i \geq \phi^j(A^*) \tag{23}$$

Since consumer *i* will prefer a smaller, to a larger, T^i, constraint (22) will be binding and (23) can be rewritten as:

$$\phi^i(A^j) + \phi^j(A^j) - \phi^j(A^*) \tag{24}$$

and the first order conditions for solving this are:

$$\phi^{i\prime}(A^j) = \phi^{j\prime}(A^j) \tag{25}$$

which is the condition for a social optimum. Then consumer *i* offers consumer *j* a payment of $\tilde{T} = \phi^j(A^*) - \phi^j(A^{**})$ if he/she would restrict his/her action to $A^j = A^{**}$. This offer ($A^{**}, \tilde{T}$) is accepted by consumer *j*.

The optimal level of action, A^{**}, is the same, irrespective of whether consumer *i* or *j* have "property rights". However, the distributional implications are very different: when consumer *i* has the right to be free of the externality, consumer *j* has to make a payment of $\hat{T}$ to consumer *i*; however, if consumer *j* has the right to the action, consumer *i* has to make a payment of $\tilde{T}$ to consumer *j*. If property rights are yet to be assigned, they should be assigned to the party who has to pay the higher cost to avoid the externality (i.e. avoidance costs should be met by the "least-cost avoider"): to consumer *i*, if $\hat{T} < \tilde{T}$, and to consumer *j*, if $\hat{T} > \tilde{T}$.